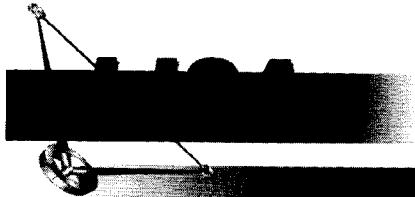


Post-processing implementation of time-delay interferometry with interpolation

**Daniel Shaddock, Brent Ware, Bob Spero and Michele
Vallisneri**

**Jet Propulsion Laboratory
California Institute of Technology
25 March 2004**

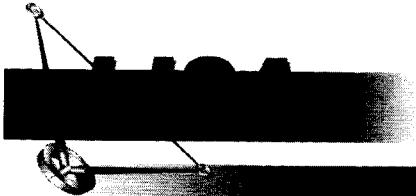
Daniel.Shaddock@jpl.nasa.gov



Fractional Delay



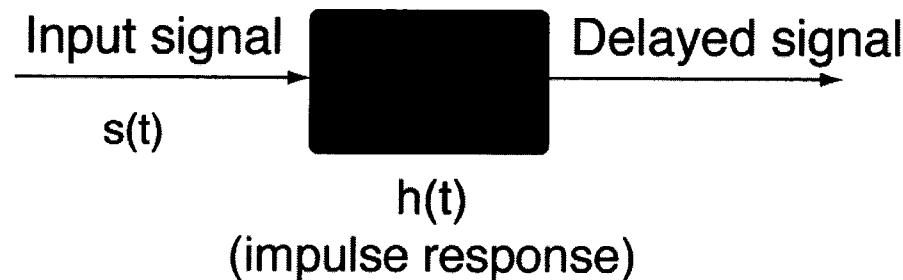
- Timing accuracy of phase measurements for time-delay interferometry (TDI) implementation: $\delta t < 50 \text{ ns}$
- Phasemeter data transmitted at 10 Hz.
- Current baseline approach is to trigger phasemeter measurements at the correct time(s)
- Alternative approach is to sample at a constant rate and interpolate in post-processing (back on Earth).
 - Previous calculations have shown this alternative approach to be not feasible (several months of continuous data needed to interpolate to required accuracy).
 - New results indicate that it is very feasible (less than 30 seconds of data needed for interpolation).



Delay by filtering



Approach: Filter signal to produce desired delay.

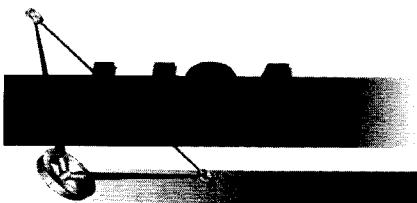


All-pass filter $h(t) * s(t) = s(t)$

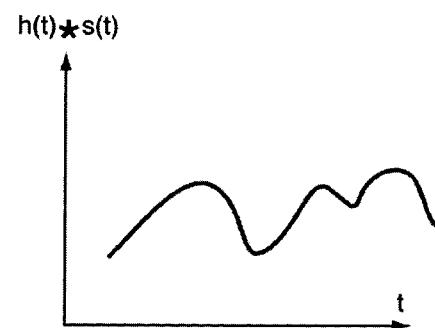
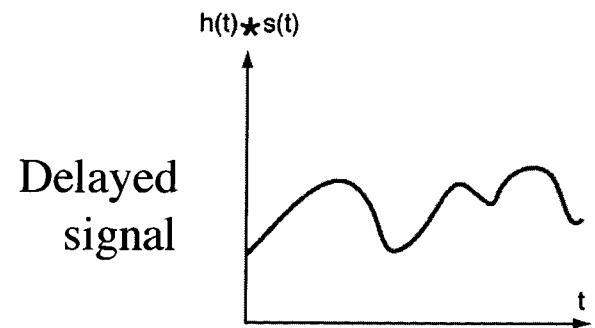
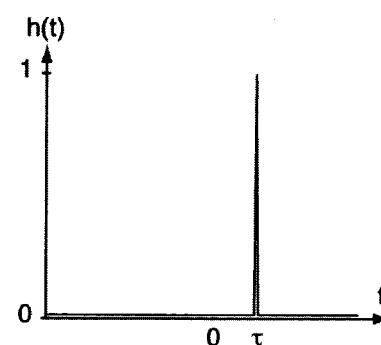
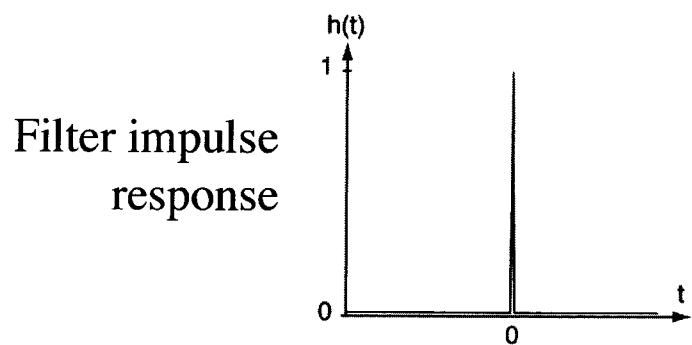
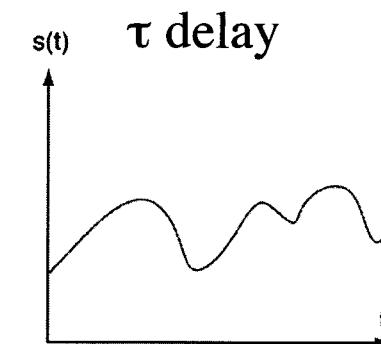
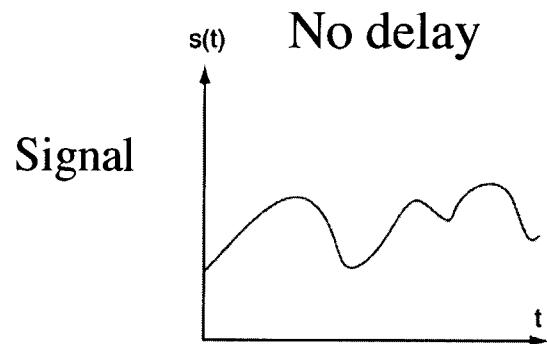
$$s(t - t_0) = h(t) * s(t - t_0)$$

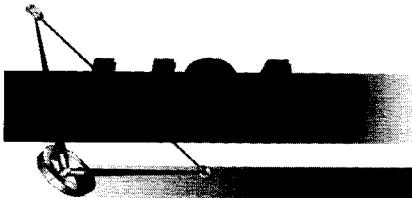
Fourier Transform $H(\omega)[S(\omega)e^{-i\omega t}] = [H(\omega)e^{-i\omega t}]S(\omega)$

→ $s(t - t_0) = h(t - t_0) * s(t)$

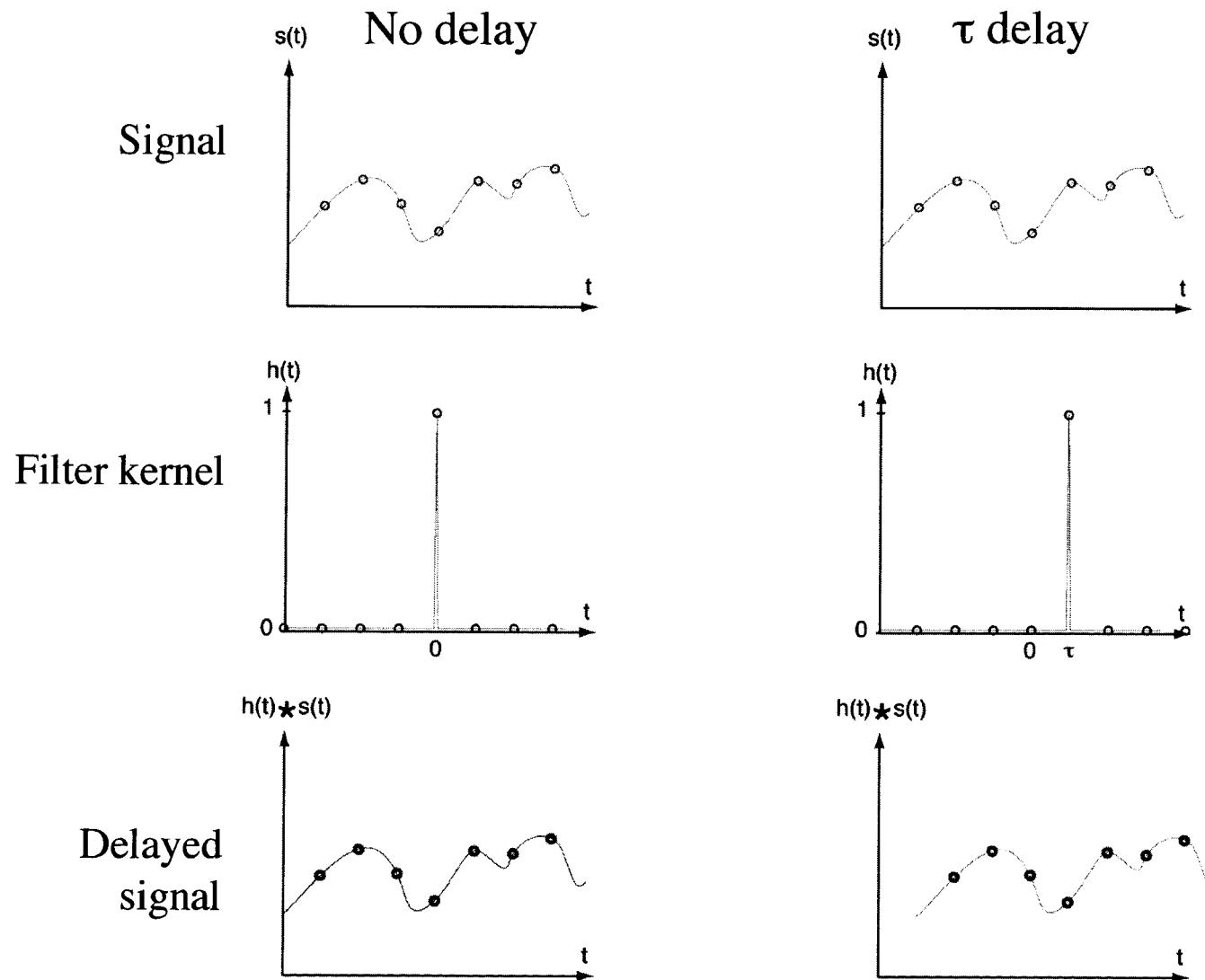


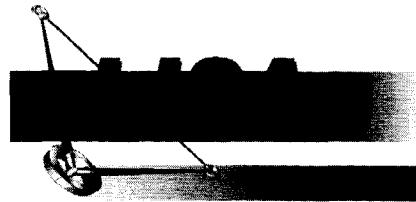
Delaying continuous signals



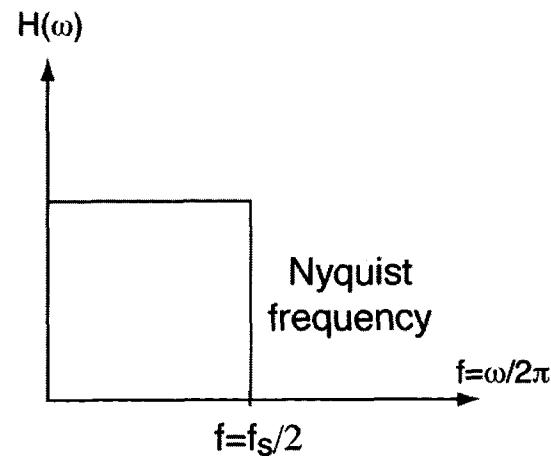
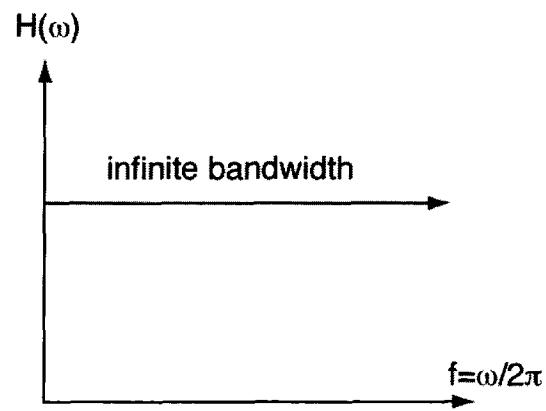
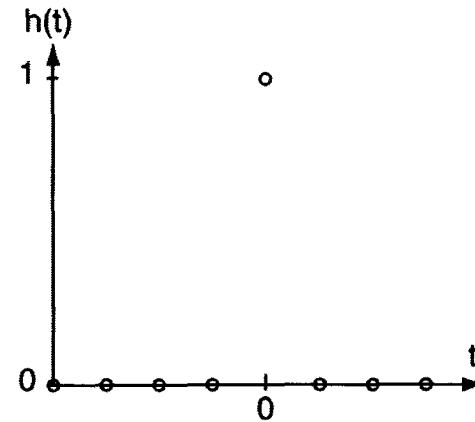
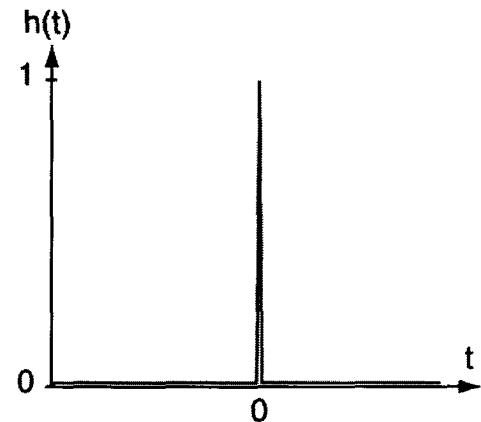


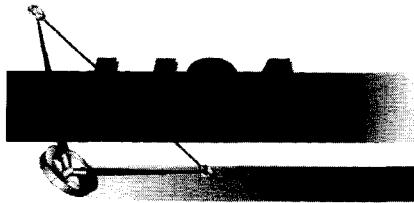
Delaying sampled signals



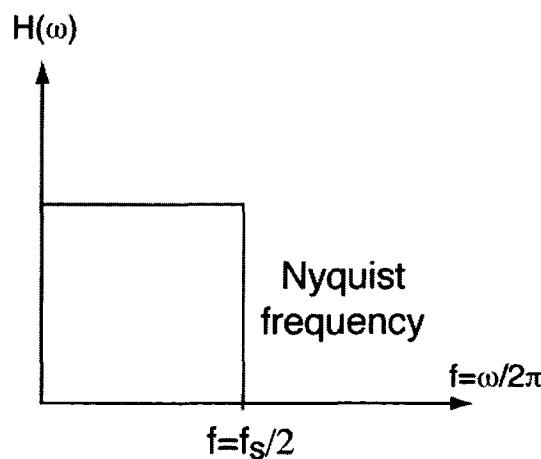
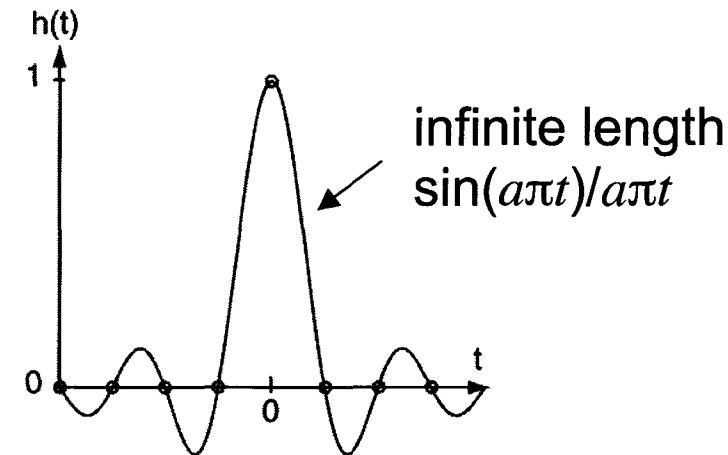
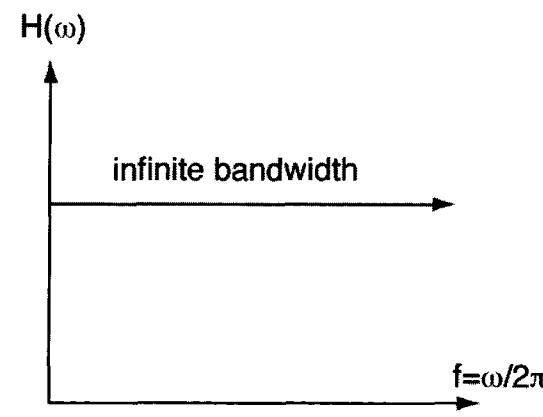
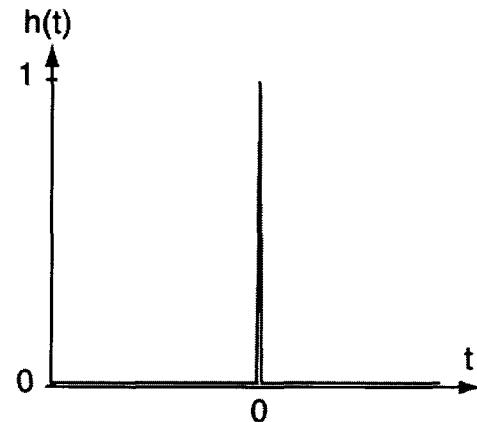


Continuous and sampled data



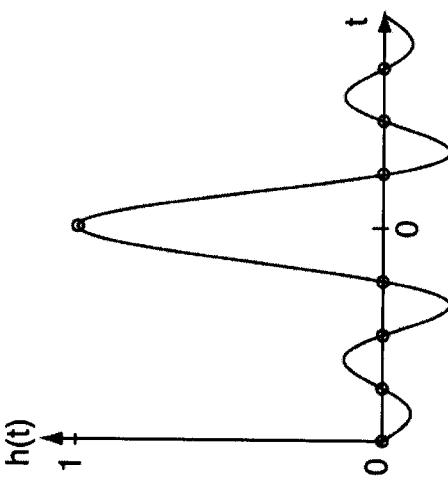


Continuous and sampled data



Fractional delay

- Interpolate data by analytically interpolating the expression for the filter kernel.



$h(t)$

1

0

-1

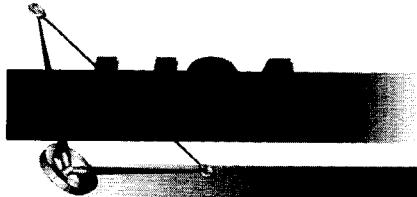
0

t

Fractional Delay=0

Fractional Delay= π

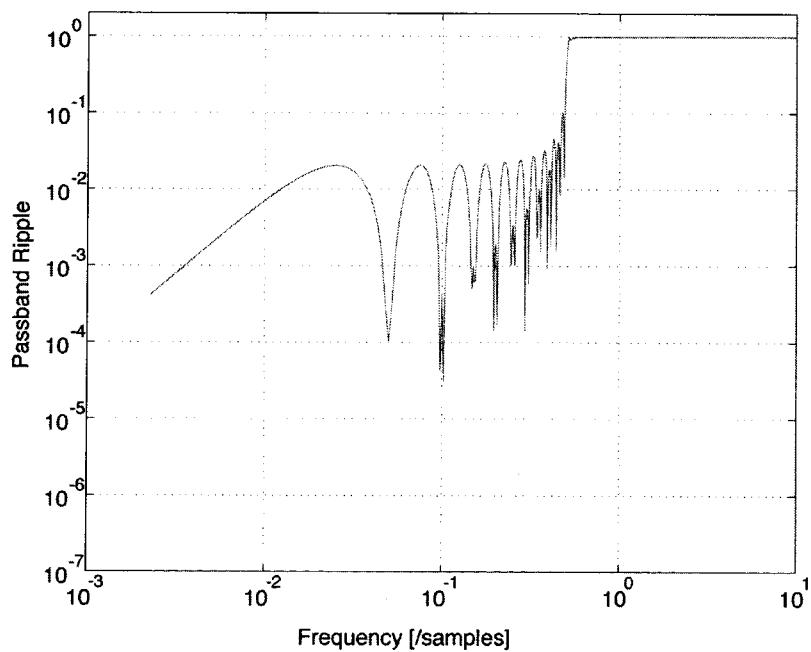
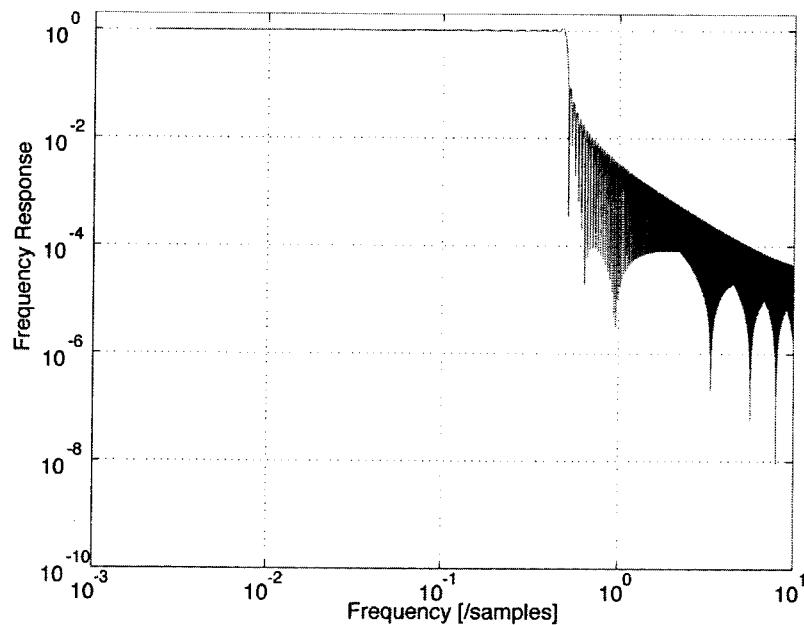
- Note that as this is a post-processing technique we are free to use a non-causal filter.
- For perfect interpolation the length of the filter kernel (and amount of data required) is infinite.



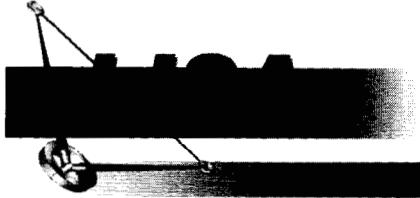
Finite length sinc



- Truncating sinc changes the frequency response of the filter.
 - Adds ripple in the pass-band.
 - Finite stop-band suppression.
 - Not suitable for use in LISA (requires several months of continuous data) [1].



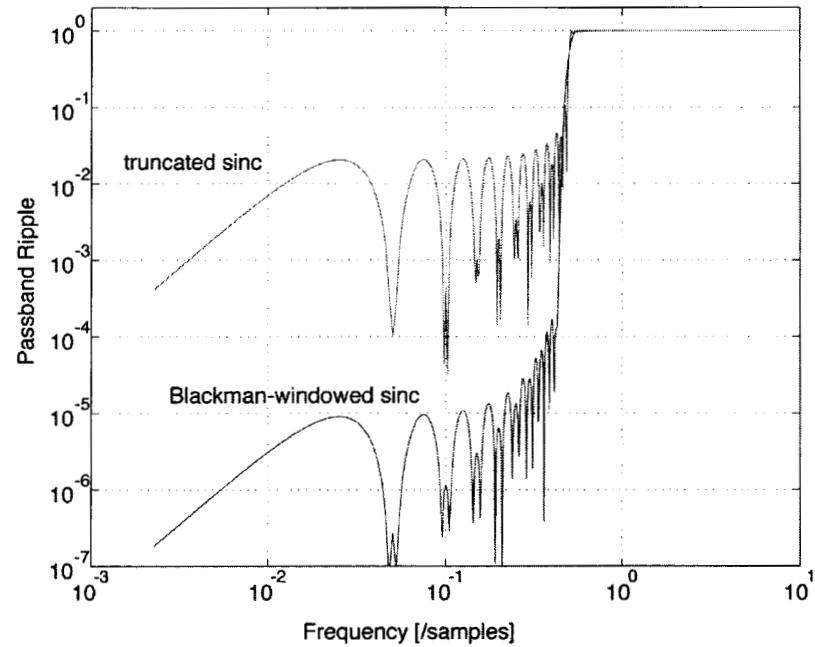
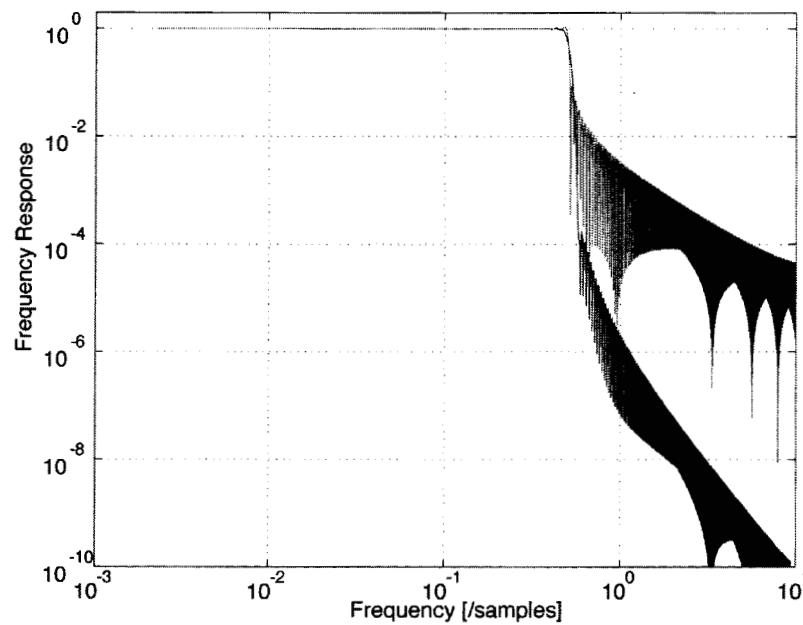
[1] M. Tinto, D.A. Shaddock, J. Sylvestre, J.W. Armstrong, "Implementation of time-delay interferometry" Phys. Rev. D **67** 122003 (2003).

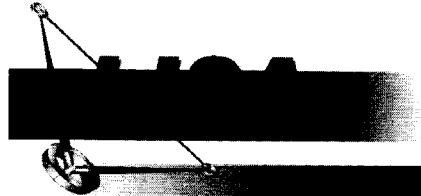


Windowed sinc



- Windowing the sinc function provides significantly better performance (many orders of magnitude).
 - Provides significantly better pass-band ripple and stop-band attenuation.
 - Approximately 30 seconds of data required to interpolate.

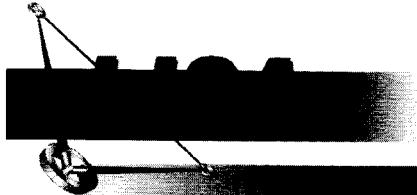




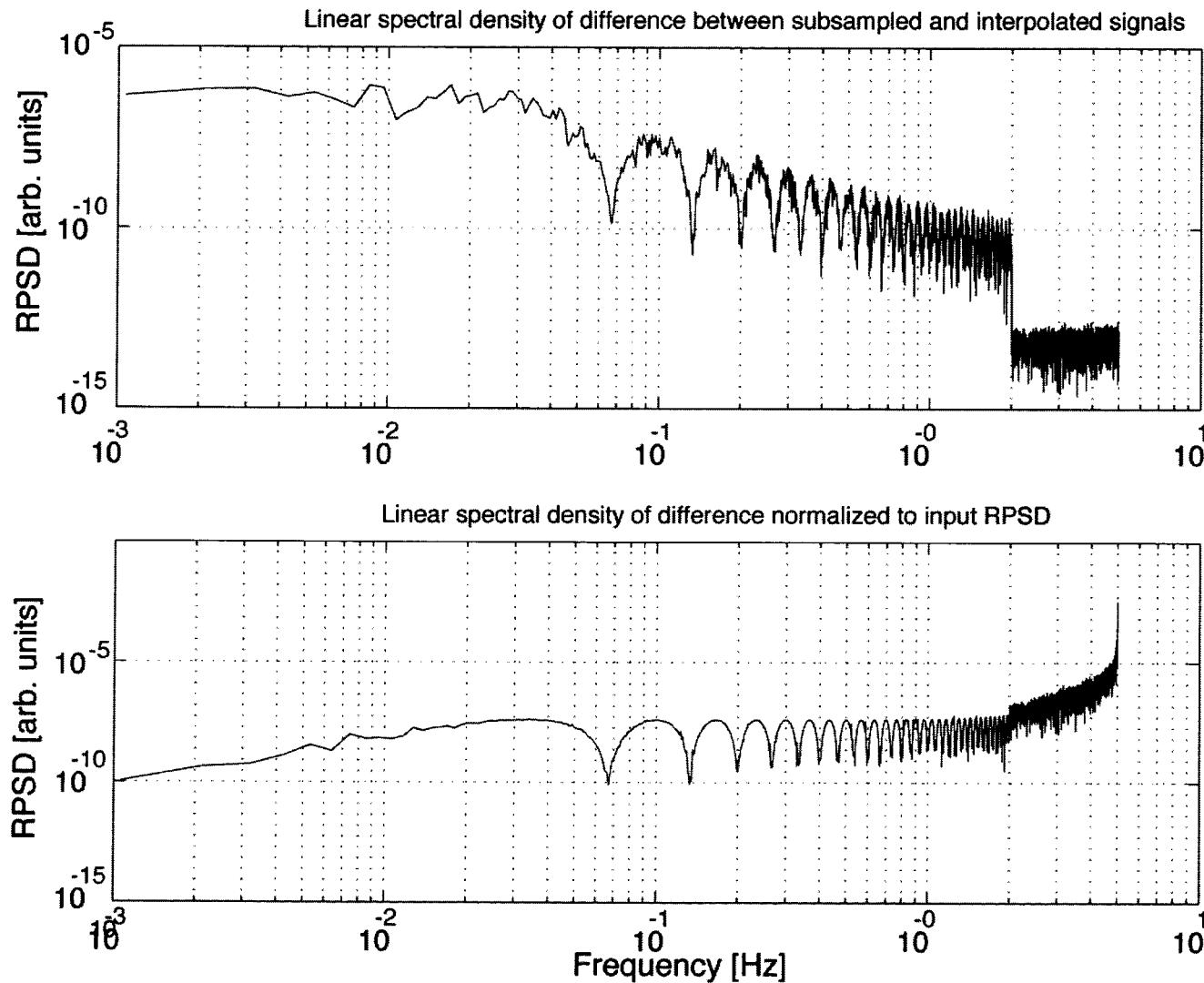
Testing and Simulation

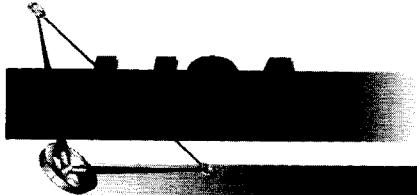


- Tested by interpolated a variety of functions
 - sinusoidal signals
 - band limited white noise
 - band limited 1/f noise
 - blind tests (thanks to John Armstrong) of both band limited and non band limited signals.



Results

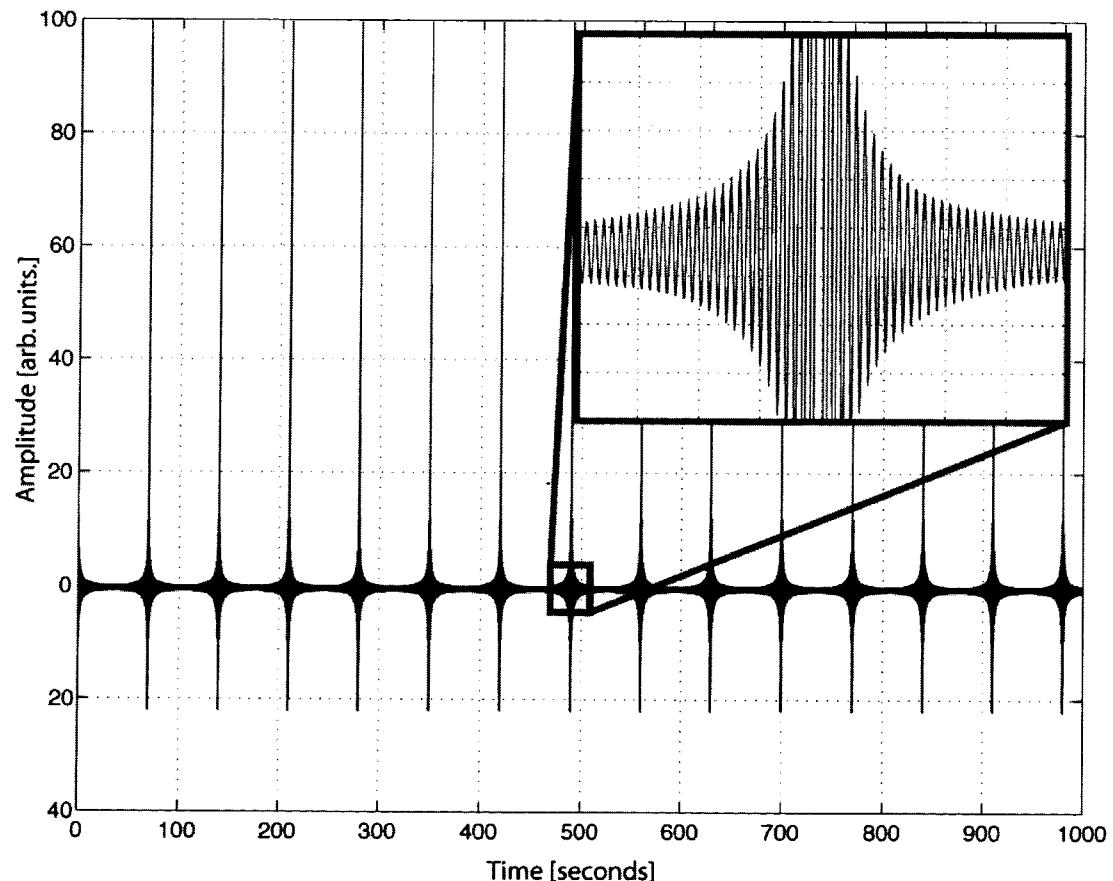


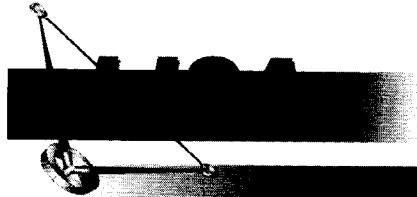


Blind tests

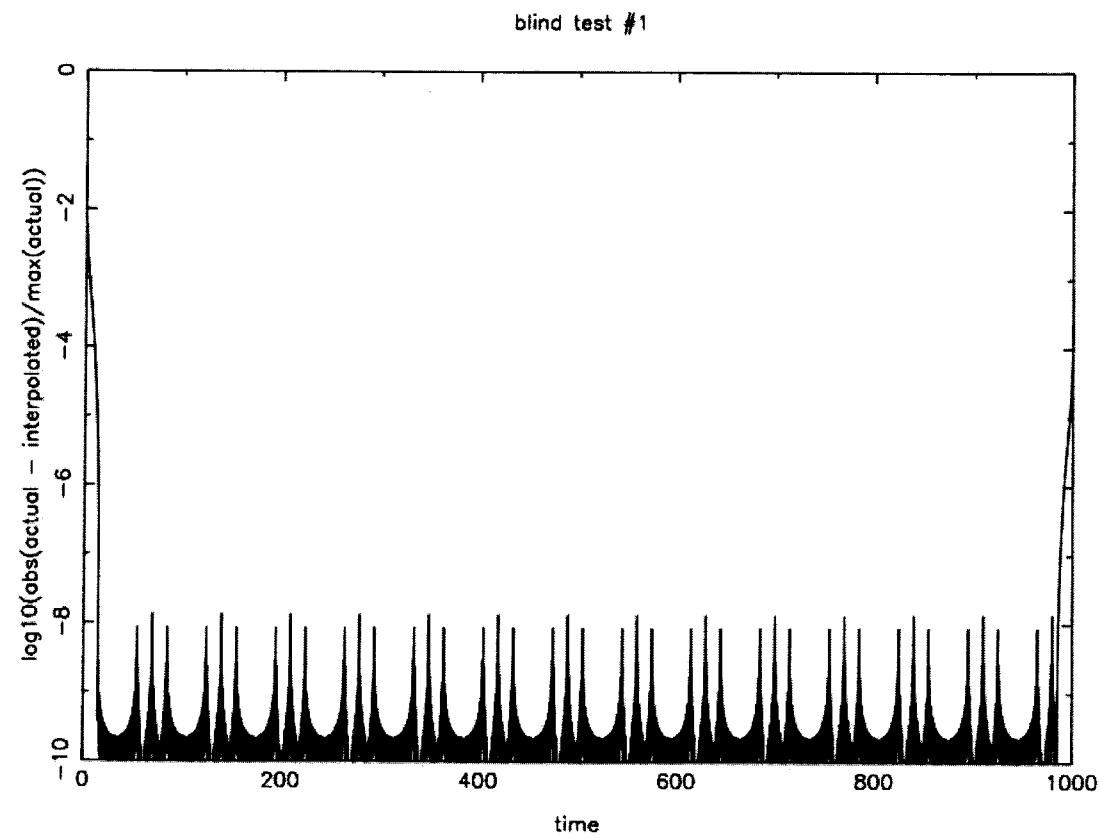


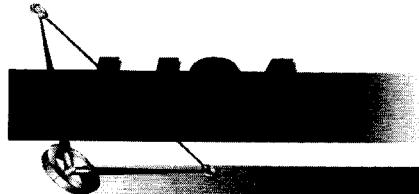
Signal generated by John Armstrong, emailed, interpolated (using windowed sinc), returned to John and subtracted from known value of signal at that time.



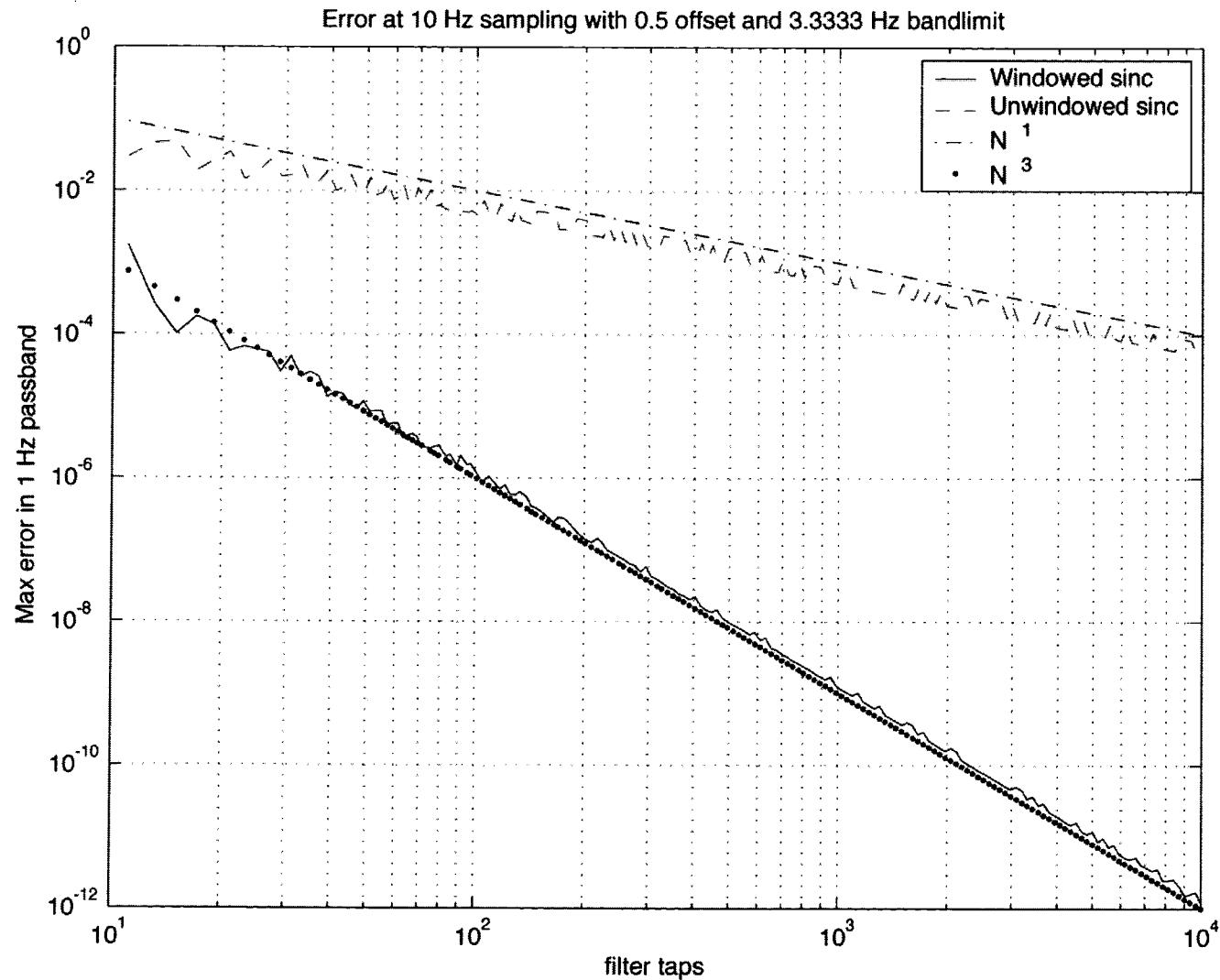


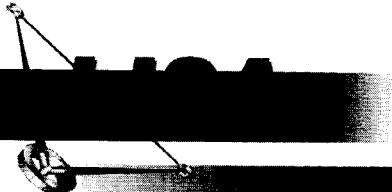
Blind test results





Error vs filter length





IIR or FIR filters



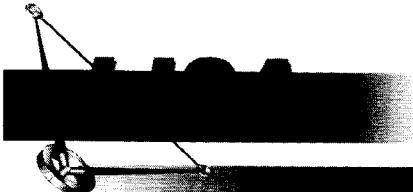
- Windowed-sinc is a finite impulse response (FIR) filter.
 - Output of filter, $y(n)$ is the weighted sum of filter's inputs $x(n)$.

$$y(n) = b(1)x(n) + b(2)x(n-1) + \dots + b(nb+1)x(n-nb)$$

- Infinite impulse response filters can also be used.
 - Output of filter is the weighted sum of current and previous inputs AND the filters previous outputs.

$$y(n) = b(1)x(n) + b(2)x(n-1) + \dots + b(nb+1)x(n-nb) \\ - a(2)y(n-1) - \dots - a(na+1)y(n-na)$$

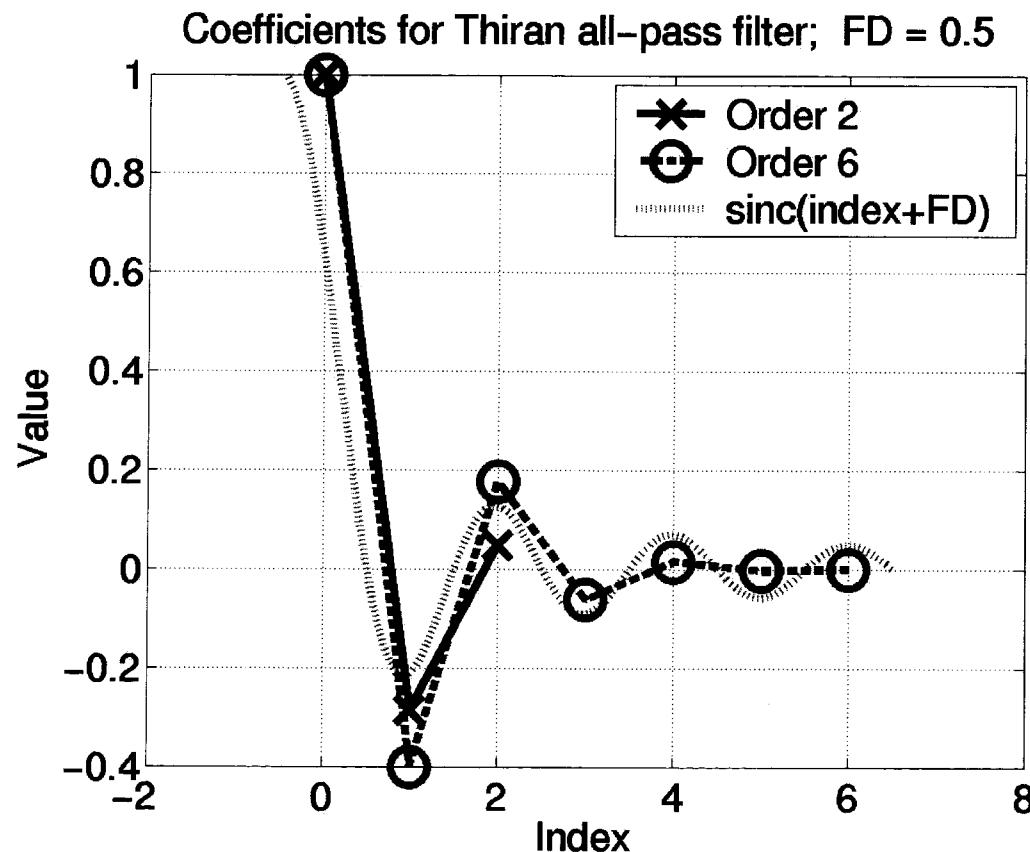
[2] T.I Laakso et al "Splitting the unit delay. Tools for fractional delay filter design." IEEE Signal Processing Magazine, January 1996.



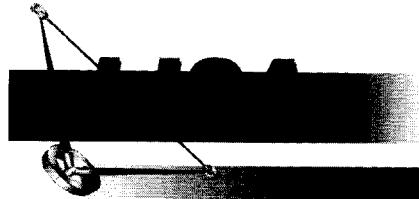
Thiran all-pass Filter



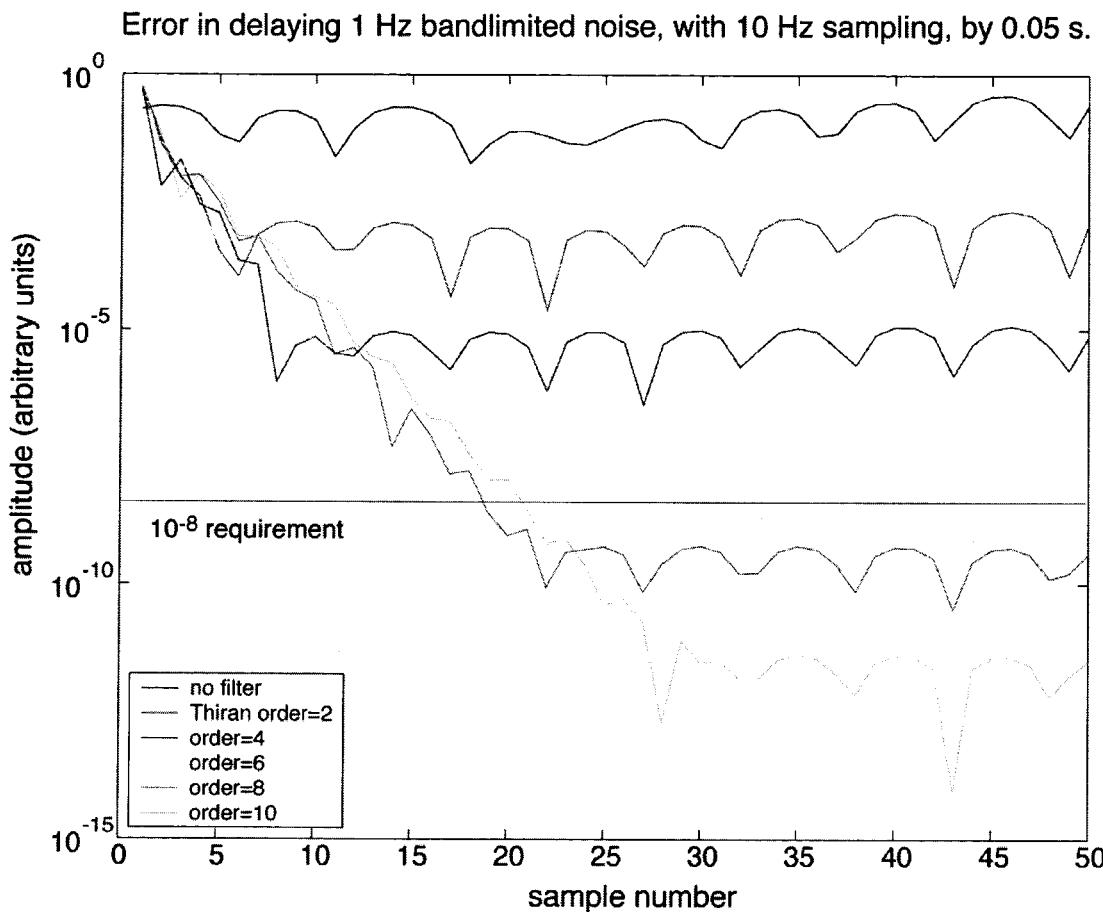
- Thiran filter [2] was tested by Bob Spero and Michele Vallisneri and looks very promising.



<http://www.acoustics.hut.fi/software/fdtools/>



Thiran all-pass filter



Preliminary tests indicate only 2 or 3 seconds of data needed for Thiran filter.

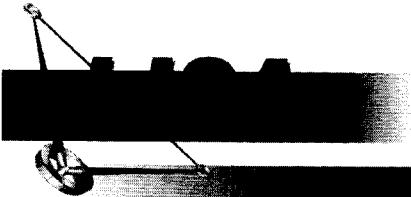
Sampling Times of Raw Time Series (Second-Generation TDI*)



	α_1	X_1	ζ_1
S_{21}	0/312	0/2'2/33'2'2/2'22'233'	132/11'
S_{31}	0/21'3'	0/33'/33'33'2'2/2'233'	1'2'3'/1'1
S_{12}	12/122'1'3'	3'/3'2'2/3'33'2'2/3'2'22'233'	22'3'/21
S_{32}	3'/3'3312	--	22'3'/21
S_{13}	1'3'/1'3'312	2/233'/22'233'/233'33'2'2	3'32/3'1'
S_{23}	2/22'1'3'	--	3'32/3'1'
$T_{21}-T_{31}$	0/2'1'3'312	0/2'2/33'/33'33'2'2/2'22'233'/2'233'33'2'2	22'33'/11'
$T_{32}-T_{12}$	3'/12/3'312/122'1'3'	--	22'3'/21/13'32/13'1'
$T_{13}-T_{23}$	2/1'3'/22'1'3'/1'3'312	--	22'3'1'/211'/233'/3'1'
r_{21}	0/123	0/22	1/11/123
r_{31}	0/123	0/33	1/11/123
r_{12}	12/123	3/223	2/21/223
r_{32}	3/123	--	2/21/223
r_{13}	13/123	2/233	3/31/233
r_{23}	2/123	--	3/31/233

- (a) 2-nd generation TDI: light times unequal in opposite directions for a given arm, due to aberration, and themselves functions of time; Shaddock et al. gr-qc/307080 (2003); Tinto et al. gr-qc/0310017 (2003) Eq. (43), Tinto et al. PRD 65 082003 (2002) Eq. (27) (with notation change)
- (b) 0/2'3 means sample now and at time delayed by $L_2(t - L_3(t)) + L_3(t)$
i.e., the order of the indices matter in the evaluation to cancel laser noise

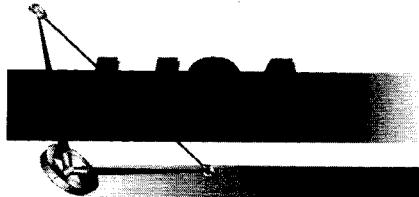
Slide extracted from J.W. Armstrong, JPL P/L meeting presentation October 2003.



Conclusions



- Post-processing interpolation (fractional delay) of phasemeter data is feasible with less than 30 seconds of data.
- Simplifies operation.
 - Eliminates the need for nanosecond-scale triggering of the phasemeters.
 - Phasemeters sample at a constant rate time tagged by local clock.
 - No arm length knowledge needed on-board (by phasemeters, payload computer.)
 - S/C clocks do not need to be synchronized in real-time. A correction can be applied in post-processing.
- Simplifies hardware.
 - Fewer phasemeter channels (factor of ~10 reduction).
 - Reduction in, or removal of, inter-spacecraft communications (one data stream per phasemeter).



Conclusions



- Reduction in data may allow time-delay interferometry combinations to be completely constructed on Earth.
 - Gives scientists access to raw data.
 - Allows flexibility to change algorithms construct 1st or 2nd (or 3rd?) generation TDI variables. $X, Y, Z, X_1, X_2, X_3, \zeta, \alpha, \beta, \gamma, \alpha_1, \alpha_2, \alpha_3, A, E, T$, etc.
 - Increase in data transmitted to Earth (signals transmitted to Earth require larger dynamic range). TBD.
- Potentially improved frequency noise suppression.
 - Estimates of arm length and clock synchronization/correction may be improved by use of post-processing techniques (e.g. auto-correlation) and adjusted after the fact.